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**TECHNICAL REPORT RR-CR-80-1** 

**DETECTABILITY OF OBJECTS IN SPECKLE** 

Adam Kozma Environmental Research Institute of Michigan Ann Arbor, Michigan

Prepared for:

Research Directorate
US Army Missile Laboratory

October 1979



U.S. ARMY MISSILE COMMAND

Redetone Arsenal, Alabama 35809

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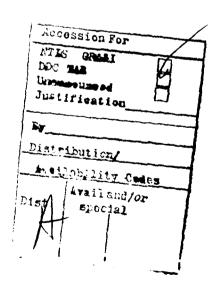
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Theoretical work based on the statistical technique applied to speckle noise in coherent images is extending the statistical technique applied to determination of probability of the statistical technique for low and medium contrast objects immersed in specific contracts and medium contracts objects immersed in specific contracts.	

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#### I. INTRODUCTION

When diffuse objects are illuminated with coherent radiation, the appearance of the image is degraded by the presence of speckle. The effect of the speckle is to superimpose a noise-like structure which masks the spatial information present in the image.

Considerable work has been done in characterizing the speckle over the last few years [1]. Both theoretical and experimental work has revealed new ideas which have enhanced our understanding of the speckle phenomena.

The purpose of this work is to extend the statistical techniques of hypothesis testing first used by Dainty [2] to study speckle and to apply these extended techniques to the data generated in the studies conducted by Guenther et al. [3,4].

Dainty used simple statistical detection theory to study the problem of detecting small images immersed in a background of laser-produced speckle. He assumed that the detection device was a flying-spot scanner and showed that the size and transmittance of the scanning aperture was best determined by statistical considerations and not by signal-to-noise (SNR) criteria. He also showed that the typical detection problem of locating opaque disks in a speckle background requires that the geometrical image diameter be at least four times the Rayleigh resolution limit of the lens used to form the image for reliable detection. The work by Dainty was theoretical; no experimental data were presented to test his theory.

On the other hand, the work in References 3 and 4 was largely experimental with estimates of detection determined empirically. In that work, a special test target pattermed after one first used by Rose is used. The target has disks of varying transmissivity and diameter which are imaged in the presence of speckle. The transmittance of the disks varies from 0.88 to 0.012, while the size varies from about two times the speckle size to about 14 times the speckle size. The test target is imaged under various illumination conditions.

In the following we will present, in a clear way, the theory first proposed by Dainty and show how this theory can apply to the experimental work of Guenther et al.

#### II. TESTING OF SIMPLE HYPOTHESIS [5]

In hypothesis testing, an observer measures a single quantity X and on the basis of this measurement chooses between two hypotheses  ${\rm H}_0$  or  ${\rm H}_1$ . If X always was equal to  ${\rm a}_0$  when the hypothesis  ${\rm H}_0$  was true and to  ${\rm a}_1$  when  ${\rm H}_1$  was true, there would be no need for statistical hypothesis testing. However, because we may make errors in measurement or we have factors operating which we do not understand, the observation quantity X is a random variable that we can describe statistically by giving a probability density function  $p_0\left(X\right)$  or  $p_1\left(X\right)$  if  ${\rm H}_0$  or  ${\rm H}_1$  applies. The hypotheses are called simple if both  $p_0\left(X\right)$  and  $p_1\left(X\right)$  are completely known and they do not depend on unknown parameters.

A strategy must be adopted by the observer which assigns one hypothesis or the other to each observable outcome. This strategy divides the possible values of X into two ranges,  $R_0$  and  $R_1$ , so that if X lies in  $R_0$ ,  $H_0$  is chosen and if X lies in  $R_1$ ,  $H_1$  is chosen. The regions should be so determined as to get the best average results in the experiment.

A simple example of this statistical test is to choose between two hypotheses:

Ho: rain tomorrow

H1: no rain tomorrow

A possible way of obtaining such a decision is to determine the average rate of change of barometric pressure during the past 24 hours.

Suppose that

 $p_0(X)$  is the p.d.f. of rates X on days before rain =

$$\left(2\pi\sigma^2\right)^{-\frac{1}{2}} \exp \left[-\left(X-a_0\right)^2/2\sigma^2\right]$$

 $p_1(X)$  is the p.d.f. of rates X on days before no

rain =

$$\left(2\pi\sigma^{2}\right)^{-\frac{1}{2}} \exp \left[-(X - a_{1})^{2}/2\sigma^{2}\right]$$

where  $a_0 < 0 < a_1$ .

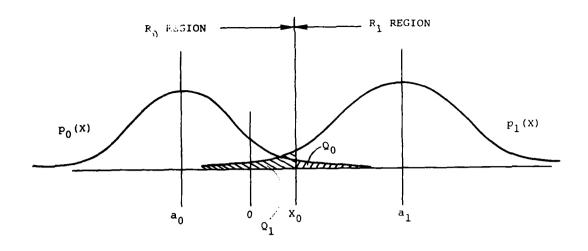


Figure 1. Probability density functions under hypotheses  ${\rm H}_{\rm 0}$  and  ${\rm H}_{\rm 1}$ .

These p.d.f.'s are shown in Figure 1. We can see that on days before rain the barometer fell with an average rate a which is negative, while on days before no rain the average rate is positive because the barometer was rising. In addition to the above data contained in the p.d.f., suppose we know the fraction  $\zeta$  of rainy days that occurred in the past. This number  $\zeta$  can be called the prior probability of rain and  $(1-\zeta)$  the prior probability of no rain.

As we can see in Figure 1, the p.d.f.'s overlap; thus, no matter how we choose regions  $R_0$  and  $R_1$ , we will occasionally make a wrong decision. In fact, the probability of choosing  $H_1$  when  $H_0$  is true, so-called error of the first kind, is

$$Q_0 = \int_{X_0}^{\infty} p_0(x) dx,$$

shown as the crosshatched area under the  $p_0\left(X\right)$  curve. The probability of choosing  $H_0$  when  $H_1$  is true, an error of the second kind, is

$$Q_1 = \int_{-\infty}^{X_0} p_1(X) dx$$
.

The value of X0 which the observer will choose depends on how much wrong decisions cost. Support that  $C_0$  is the cost of an error of the first kind,  $C_1$  is the cost of an error of the second kind, and  $C_0Q_0$  and  $C_1Q_1$  are called the risk associated with  $H_0$  and  $H_1$ , respectively. Then the average risk of each decision is

$$\begin{split} \widetilde{C}(x_0) &= \zeta C_0 Q_0 + (1 - \zeta) C_1 Q_1 \\ &= \zeta C_0 \int_{X_0}^{\infty} p_0(x) dx + (1 - \zeta) C_1 \int_{-\infty}^{X_0} p_1(x) dx. \end{split}$$

We can minimize the above by differentiating with respect to  $\mathbf{X}_0$  and putting the result equal to zero. Thus,

$$\frac{p_1(X_0)}{p_0(X_0)} = \frac{\zeta C_0}{(1 - \zeta)C_1} = \Lambda_0$$

which yields

$$x_0 = (a_0 + a_1)/2 + \frac{\sigma^2}{a_1 - a_0} \ln \frac{\zeta C_0}{(1 - \zeta)C_1}$$
.

Note that if  $\zeta = 1/2$  and  $C_0 = C_1$ , we get  $X_0 = (a_0 + a_1)/2$ .

#### III. BAYES CRITERION

The Bayes criterion is a rule which allows the decision strategy to be picked so as to minimize the average risk.

There are four types of costs possible which can be arrayed in a cost matrix C.

$$c = \begin{bmatrix} c_{00} & c_{01} \\ c_{10} & c_{11} \end{bmatrix}$$

where  $C_{ij}$  is the cost of choosing  $H_i$  when  $H_j$  is true (i, j = 0, l). The relative cost of error of the first kind is  $C_{10}$  -  $C_{00}$  and of the second kind  $C_{01}$  -  $C_{11}$ , with both relative costs being positive. In the example,

$$C = \begin{bmatrix} 0 & C_1 \\ C_0 & 0 \end{bmatrix}.$$

In the general case where elements of the cost matrix are not zero, we have the average per decision as

$$\overline{C} = \zeta \left[ c_{00} \int_{-\infty}^{X_0} p_0(x) dx + c_{10} \int_{X_0}^{\infty} p_0(x) dx \right]$$

$$+ (1 - \zeta) \left[ c_{01} \int_{-\infty}^{X_0} p_1(x) dx + c_{11} \int_{X_0}^{\infty} p_1(x) dx \right].$$

It can be shown that the average risk is minimum when  $\mathbf{X}_0$  is chosen so that, given the likelihood ratio

$$\wedge(x) = p_1(x)/p_0(x),$$

 $R_0$  consists of values of X for which  $\Lambda(X)$  <  $\Lambda_0$  and  $R_1$  consists of values of X for which  $\Lambda(X)$  >  $\Lambda_0$  where  $\Lambda_0$  is

$$\Lambda_0 = \frac{\zeta (C_{10} - C_{00})}{(1 - \zeta) (C_{01} - C_{11})}.$$

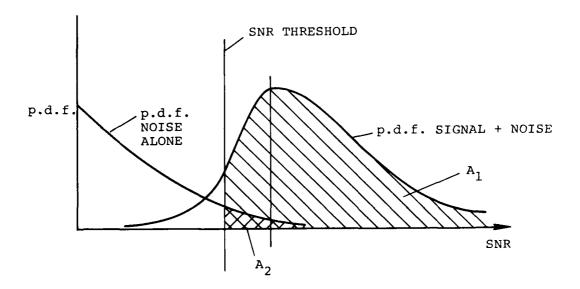
If  $\zeta$ , the prior probability is known and values can be assigned to  $C_{ij}$  and the threshold  $X_0$  dividing regions  $R_0$  and  $R_1$  can be found which minimizes the average risk for each decision. For the general cost of the detection of signals in noise, it is often the case that we cannot reasonably assign risks nor can we know prior probabilities; the position of the threshold is set from experiment by trial and error.

### IV. SIGNALS IN NOISE

The theory outlined above is often used in the design of communications and radar systems. There, often

the problem is to determine whether a signal and additive noise are being observed or whether noise alone is present.

In this case, the situation is illustrated in Figure 2. Here the crosshatched area A<sub>1</sub> under the p.d.f. curve of signal + noise gives the probability of detection while the doubly crosshatched area A<sub>2</sub> under the p.d.f. curve of noise alone is the probability of a false alarm. The problem here is determining the proper signal-to-noise ratio and threshold to give an adequate probability of detection while keeping the probability of a false alarm properly low. The SNR, which determines the relative position of the p.d.f. of signal + noise to the p.d.f. of noise alone, along with the proper choice of threshold, allows obtaining proper values of detection and false alarms.



 $A_1$ =probability of detection  $A_2$ =probability of false alarm

Figure 2. Probability of noise alone and signal + noise illustrating the process of threshold detection.

### V. DISK IMAGES IN SPECKLE

Suppose we have disk objects of intensity transmissivity  $\langle T_i \rangle$  backed by a ground glass with average transmissivity of  $\langle T_S \rangle$  where we assume  $\langle T_S \rangle \rangle \langle T_i \rangle$ . The diameter of the disk is given by d where d  $\geq$  d<sub>S</sub>. The quantity

ds is the approximate size of the speckle as determined by the Rayleigh criterion. That is,  $d_S = \frac{\lambda F}{D}$ , where F is the focal length of the lens and  $\lambda$  is the wavelength of the light. Figure 3 illustrates this situation.

Suppose further that we place a photocell at plane  $P_3$  with a diameter of  $d_p$ . This photocell is used to measure the amount of light which is present at this plane. We can use the voltage reading from the photocell to determine if a disk is present or not. The intensity measured by the photocell is

$$I_{p} = \int_{-\infty}^{\infty} I(x,y) B(x,y) dxdy$$
 (1)

where I(x,y) is the intensity distribution at  $P_3$  and

$$B(x,y) = 1 \text{ for } x^2 + y^2 \le d_p$$

$$= 0 \text{ otherwise;}$$
(2)

 $I\left(x,y\right)$  can either be the intensity due to the presence of a disk in which case its average value as a function of x and y is given by

$$\langle I(x,y) \rangle_{d} = |0_{A}(x,y)|^{2} * |K(x,y)|^{2}$$
 (3)

where

$$|0_{A}(x,y)|^{2} = I \langle T_{i} \rangle \langle T_{S} \rangle \text{ for } x^{2} + y^{2} \leq d^{2}$$

$$= I \langle T_{S} \rangle \text{ otherwise;}$$
(4)

I is the uniform intensity of the laser illumination and  $|K(x,y)|^2$  is the intensity point spread function of the optical system given by

$$|K(x,y)|^2 = \left(\frac{kD^2}{8F}\right)^2 \left[2 \frac{J_1 \left(\frac{kDr_0}{2F}\right)}{\frac{kDr_0}{2F}}\right]$$
 (5)

where  $k=2\pi/\lambda$ ,  $J_1(X)$  is the first-order Bessel function and  $r_0^2=x^2+y^2$ . I(x,y) will vary around <I(x,y)> due to the presence of speckle superimposed over the disk image by the ground glass.

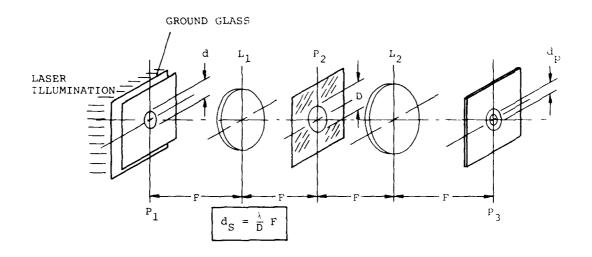


Figure 3. Imaging a dark disk with a coherent optical system in the presence of speckle noise.

If there is no disk present in plane  $P_1$ , then the intensity I(x,y) is the intensity of the imaged ground glass. The average value of this intensity is given by

$$\langle I(x,y) \rangle_{\text{nd}} = I \langle T_S \rangle.$$
 (6)

Again, I(x,y) will vary around the average  $\leq I^{\perp}$  because of the ground glass.

The p.d.f. associated with the photocell reading due to the intensity variation when no disk is present is given by  $p_0\left(I\right)$ , while the p.d.f. associated with the photocell reading when a disk is present is given by  $p_1\left(I\right)$ . The p.d.f.'s are shown in Figure 4. Generally, the p.d.f.'s will overlap and with a threshold  $I_{T}$  sometimes the wrong decision will be made whether a disk is or is not present. Two kinds of errors can be made. Errors of the first kind are the apparent detection of an image when it is not present. This false alarm probably  $\alpha$  is given by

$$\alpha = \int_{0}^{T} p_{0}(I) dI$$
 (7)

and is shown as the filled in area under the  $p_0(I)$  curve in Figure 4. The second kind of error is in not detecting an image which is actually present. This probability is given by  $(1-\beta)$  where  $\beta$  is the probability of a correct detection. The probability of a correct detection is the crosshatched area under the  $p_1(I)$  curve.

$$\beta = \int_{0}^{T} p_{1}(I) dI \qquad (8)$$

The shape of the p.d.f.'s  $p_0$  and  $p_1$  are determined by

- The statistical properties of the speckle;
- The intensity transmittance of B(x,y);
- The intensity transmittance of the ground glass and disk image;

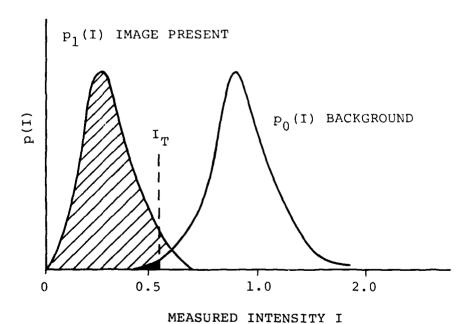


Figure 4. Single threshold detection.

and the distance between the p.d.f.'s is determined by the average value of the intensity transmittance of the disk and ground glass.

### VI. PROBABILITY DENSITY FUNCTIONS

An approximate expression for  $P_0$  and  $P_1$  [6] is

$$p(I) = \frac{\left(\frac{M}{\langle I \rangle}\right)^{M} I^{M-1} \exp\left(-\frac{MI}{\langle I \rangle}\right)}{\Gamma(M)}$$
(9)

for I > 0, zero otherwise. Where  $\Gamma(X)$  is the gamma function, M =  $(\langle I \rangle/\sigma_I)^2$ ,  $\langle I \rangle$  average value of the p.d.f. and  $\sigma_I$  is the standard deviation.

### VII. CALCULATION OF $\alpha$ AND $\beta$

In order to proceed with the calculation of  $\circ$  and  $\circ$  , we need to calculate the mean <I> and the standard deviation  $\sigma_I$  of the p.d.f.'s when an image is present and not present.

### p<sub>0</sub>(I) [No Disk Present]

$$\langle I \rangle_{nd} = I \langle T_S \rangle \tag{10}$$

$$(\sigma_{\mathbf{I}})_{\text{nd}}^{2} = \langle \mathbf{I} \rangle_{\text{nd}} \int_{-\infty}^{\infty} N_{\text{nd}}(\mathbf{u}, \mathbf{v}) | b(\mathbf{u}, \mathbf{v}) |^{2} d\mathbf{u} d\mathbf{v}$$
 (11)

where b(u, v) is the Fourier transform of B(x, y).

$$N_{nd}(u, v) = K_{nd} |F(u, v)|^2 * |F(-u, -v)|^2$$
 (12)

where F(u, v) is the aperture of the filter in plane  $P_2$ . The constant  $K_{\mbox{nd}}$  can be evaluated from the fact that

$$\int_{-\infty}^{\infty} p_0(I) dI = 1.$$
 (13)

Once  $<I>_{nd}$  and  $(\sigma_I)_{nd}$  are evaluated, they can be substituted into Equation (9) to obtain  $p_0(I)$ .

## p<sub>1</sub>(I) [Disk Present]

$$(\sigma_{\mathbf{I}})_{\mathbf{d}}^{2} = \langle \mathbf{I} \rangle_{\mathbf{d}} \iint_{-\infty}^{\infty} N_{\mathbf{d}}(\mathbf{u}, \mathbf{v}) |b(\mathbf{u}, \mathbf{v})|^{2} d\mathbf{u} d\mathbf{v}$$
 (14)

where b(u, v) is again the Fourier transform of B(x, y) and

$$N_{\bar{d}}(u, v) = K_{\bar{d}} |F(u, v)|^2 * |F(-u, -v)|^2$$
 (15)

where F is the same as above.  $K_{\mbox{\scriptsize d}}$  is evaluated as above by noting that

$$\int_{-\infty}^{\infty} p_1(I) dI = 1.$$
 (16)

When the two p.d.f.'s are found,  $\alpha$  and  $\beta$  can be calculated

$$\alpha = \int_{0}^{I_{T}} p_{0}(I) dI$$

$$\alpha = \int_{0}^{I_{T}} \frac{\left(\frac{M}{\langle I \rangle_{nd}}\right)^{M} I^{M-1} \exp\left(-\frac{MI}{\langle I \rangle_{nd}}\right)}{\Gamma(M)} dI$$
(17)

where

$$M = \frac{\langle I \rangle}{(\sigma_I)^2} \frac{2}{nd}$$

$$\beta = \int_{0}^{I_{T}} p_{1}(I) dI$$

$$\beta = \int_{0}^{T} \frac{\left[\frac{M}{\langle I \rangle_{d}}\right]^{M} I^{M-1} \exp\left[-\frac{MI}{\langle I \rangle_{d}}\right]}{\Gamma(M)} dI$$
 (18)

PHOLOGICAL SET

where

$$M = \frac{\langle I \rangle_{d}^{2}}{\langle \sigma_{I} \rangle_{d}^{2}}.$$

### VIII. CONCLUSIONS

We can note how the size of the disk, the intensity transmissivity and the size of the scanning aperture affect the calculation of  $\tau$  and z.

Parameter	$\widetilde{\sigma}$	<u> </u>
d size of disk	-	Eq. (3) <1/d
T <sub>i</sub> disk transmittance	-	Eq. (3) «I d
T <sub>S</sub> g.g. transmittance	Eq. (6) <1>nd	Eq. (3) - I - d
D size of aperture in plane P <sub>2</sub>	Eq. (6) $<$ $I$ $>$ nd Eq. (11) $(\sigma_I)$ nd	Eq. (3) I d Eq. (14) (0 <sub>1</sub> )d
dP size of scanning aperture	Eq. (1) <1> $_{\rm nd}$ Eq. (11) $(\sigma_{\rm I})_{\rm nd}$	Eq. (1) -1 d Eq. (14) (o <sub>1</sub> )d

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